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$\chi_{c2} \to \rho\rho$ and the ρ polarization in massless perturbative QCD: how to test the distribution amplitudes

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Abstract

We compute the helicity density matrix of ρ vector mesons produced in the two-body decays of polarized χ_{c2}'s in the framework of massless perturbative QCD. The χ_{c2}'s are either exclusively or inclusively produced in $p\,\bar{p}$ or $p\,p$ interactions, via quark-antiquark annihilation or gluon fusion. Our results show unambiguous significant differences depending on the choice of the ρ distribution amplitudes and allow to discriminate between different proposed models.

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I. INTRODUCTION

The general theoretical framework for the description of high energy exclusive processes within perturbative QCD has been widely discussed in the literature [1] and, although it has not yet reached the same level of reliability and variety of applications as the factorization theorem for the inclusive interactions, is expected to work at large momentum transfers and indeed several successful calculations have been performed so far in some simple cases [1,2]. The numerical results depend strongly on the internal structure of the hadrons involved which, in the exclusive cases, are described by the hadron wave functions or distribution amplitudes, analogous to the distribution functions of the inclusive case. Perturbative QCD fixes the asymptotic form of these amplitudes at $Q^2 \to \infty$ and their general evolution, but in any realistic calculation they have to be taken as phenomenological quantities, to be experimentally determined via a set of physical information and then used in other processes. This procedure has led to the success of some distribution amplitudes motivated by QCD sum rules [2–5].

Some criticism has been advanced to the validity of the whole scheme, on the ground that it necessarily involves a contribution from soft non perturbative regions [6]; such criticism has been answered by noticing that these soft contributions are suppressed by Sudakov form factors [7]. While this gives a better understanding and credibility to the theoretical model it weakens the agreement between theoretical computations, based on the QCD sum rule distribution amplitudes [2], and experiment; in fact these distribution amplitudes enhance the contribution of the soft end point regions, which are now known to be depressed by the Sudakov form factors. Moreover, some lattice estimates [8,9] and further theoretical considerations [10,11] have suggested different shapes for the distribution amplitudes, closer to the asymptotic ones; the situation is then less clear and better phenomenological studies have to be performed in order to improve our knowledge of the properties of the hadrons and to reach the possibility of making genuine predictions.

We consider here the $\chi_{c2} \to \rho\,\rho$ decay process of polarized charmonium states created in $p\,p$ or $p\,\bar{p}$ interactions and show how the observation of the polarization of the vector meson, via a measurement of its diagonal helicity density matrix elements, neatly depends on the ρ distribution amplitudes and helps in discriminating between different kinds of these quantities.

The same process has already been studied, with different purposes, in two previous papers [12,13], first in the exclusive channel $p\,\bar{p} \to \chi_{c2} \to \rho\,\rho$ [12] and then in the inclusive one $p\,p \to \chi_{c2} + X \to \rho\,\rho + X$ [13]. The goal there was that of assessing the significance of higher twist mass effects, rather than that of discussing the role of different distribution amplitudes: actually, it turned out that, independently of the chosen meson wave functions, only mass effects could give origin to non zero off-diagonal helicity density matrix elements, $\rho_{1,-1}(\rho)$ or $\rho_{1,0}(\rho)$. Any non zero measurement of even a single one of these non diagonal elements, signaling the violation of the helicity conservation rule, would be a clear signature of mass corrections.

Here we compute, within massless perturbative QCD, the diagonal element $\rho_{1,1}(\rho)$ and study its dependence on the distribution amplitudes, $\varphi(x)$; to be safe from mass corrections we exploit the results of Refs. [12] and [13] and only consider kinematical regions in which such corrections are known to be small. The variations resulting from the choice of different

distribution amplitudes cannot then be ascribed to other reasons. In the next Section we briefly recall the computations of Refs. [12] and [13] and show analytical expressions which are numerically evaluated in Section 3, which contains our results and where we also give some conclusive comments. We improve on the computations of Refs. [12] and [13] by explicitly introducing the Q^2 evolution of the distribution amplitudes and the strong coupling constant α_s.

II. COMPUTATION AND MEASUREMENT OF THE ρ HELICITY DENSITY MATRIX

The full processes we are considering are either the exclusive,

$$p\,\bar{p} \to \chi_{c2} \to \rho\,\rho\,, \tag{1}$$

or the inclusive,

$$p\,p \to \chi_{c2} + X \to \rho\,\rho + X\,, \tag{2}$$

production of a pair of ρ vector mesons with the subsequent decay

$$\rho \to \pi\,\pi\,, \tag{3}$$

and the quantity experimentally observed is the angular distribution of either one of the pions in the helicity rest frame of the decaying ρ [14]. In the inclusive case (2) the two initial particles need not be protons and might be other hadrons.

The pion angular distribution depends on the spin state of the ρ via the elements of its helicity density matrix $\rho_{\lambda,\lambda'}(\rho)$,

$$\begin{aligned} W(\Theta,\Phi) = \frac{3}{4\pi}[&\rho_{0,0}\cos^2\Theta + (\rho_{1,1}-\rho_{1,-1})\sin^2\Theta\cos^2\Phi \\ &+ (\rho_{1,1}+\rho_{1,-1})\sin^2\Theta\sin^2\Phi \\ &- \sqrt{2}(\mathrm{Re}\,\rho_{1,0})\sin 2\Theta\cos\Phi]\,, \end{aligned} \tag{4}$$

where Θ and Φ are, respectively, the polar and azimuthal angles of the pion as it emerges from the decay of the ρ, in the ρ helicity rest frame. By integrating Eq. (4) over Φ or Θ one has respectively the polar and azimuthal distributions

$$W(\Theta) = \frac{3}{2}[\rho_{0,0} + (\rho_{1,1}-\rho_{0,0})\sin^2\Theta]\,, \tag{5}$$

$$W(\Phi) = \frac{1}{2\pi}[1 - 2\rho_{1,-1} + 4\rho_{1,-1}\sin^2\Phi]\,. \tag{6}$$

Measurements of the above angular distributions yield direct information on $\rho_{\lambda,\lambda'}(\rho)$.

On the other hand, the vector meson helicity density matrix $\rho(\rho)$ can be computed in perturbative QCD [1,2]:

$$\rho_{\lambda,\lambda'}(\rho;\theta,\phi) = \frac{1}{N}\sum_{\mu,M,M'} A_{\lambda,\mu;M}A^*_{\lambda',\mu;M'}\,\hat{\rho}_{M,M'}(\chi_{c2}) \tag{7}$$

where N is the normalization factor such that $\mathrm{Tr}[\ \rho\ ] = 1$,

$$N(\theta, \phi) = \sum_{\lambda,\mu,M,M'} A_{\lambda,\mu;M}(\theta,\phi) A^*_{\lambda,\mu;M'}(\theta,\phi)\, \hat{\rho}_{M,M'}\,. \tag{8}$$

The $A_{\lambda,\mu;M}(\theta,\phi)$'s are the helicity amplitudes for the decay of a χ_{c2} at rest, with spin third component $J_z = M$, into two ρ vector mesons at angles θ, ϕ and $\pi - \theta, \pi + \phi$ and helicities λ and μ respectively, $\chi_{c2}(J_z = M) \to \rho(\lambda)\, \rho(\mu)$; $\hat{\rho}_{M,M'}(\chi_{c2})$ is the spin density matrix of the decaying charmonium state.

Explicit expressions of the helicity decay amplitudes $A_{\lambda,\mu;M}$ can be found in Ref. [12], computed within perturbative QCD in the general case in which (constituent) quark masses are taken into account; the massless case, which we consider here as explained in the Introduction, is simply obtained by setting $\epsilon = 0$ in the expressions of Ref. [12]. These decay amplitudes contain the ρ distribution amplitudes, $\varphi(x)$.

In order to compute the ρ density matrix via Eqs. (7) and (8) we still have to specify the spin state of the χ_{c2}; to do so we treat separately the exclusive (1) and inclusive (2) production processes. The charmonium state is produced in the exclusive $p\,\bar{p}$ channel via quark-antiquark annihilations and the helicity conservation rule of massless perturbative QCD entails [12]:

$$\begin{aligned} \hat{\rho}^{ex}_{M,M'} &= 0 \qquad\qquad M \neq M' \\ \hat{\rho}^{ex}_{0,0} &= \hat{\rho}^{ex}_{2,2} = \hat{\rho}^{ex}_{-2,-2} = 0 \\ \hat{\rho}^{ex}_{1,1} &= \hat{\rho}^{ex}_{-1,-1} = 1/2\,. \end{aligned} \tag{9}$$

In the inclusive channel (2) the χ_{c2} is produced dominantly via two gluon fusion and, assuming a non relativistic wave function for the charmonium state, one finds [13]:

$$\begin{aligned} \hat{\rho}^{in}_{M,M'} &= 0 \qquad\qquad M \neq M' \\ \hat{\rho}^{in}_{0,0} &= \hat{\rho}^{in}_{1,1} = \hat{\rho}^{in}_{-1,-1} = 0 \\ \hat{\rho}^{in}_{2,2} &= \hat{\rho}^{in}_{-2,-2} = 1/2\,. \end{aligned} \tag{10}$$

By exploiting Eqs. (7)-(10) and the analytical results of Ref. [12] one derives for the helicity density matrix of the ρ meson, produced either via the exclusive process (1) or the inclusive one (2):

$$\begin{aligned} \rho_{\lambda,\lambda'} &= 0 \qquad\qquad \lambda \neq \lambda' \\ \rho_{0,0} &= 1 - 2\rho_{1,1} \\ \rho_{1,1} &= \rho_{-1,-1} = \frac{1}{2}\, \frac{1}{1 + 3\dfrac{|\tilde{A}_{0,0}|^2}{|\tilde{A}_{1,-1}|^2} F(\theta)} \end{aligned} \tag{11}$$

where the reduced amplitudes $\tilde{A}_{\lambda,\lambda'}$ are given in Eqs. (2.10, 11) and (2.14, 15) of Ref. [12] and do not depend on the ρ production angles θ and ϕ, but do depend on the distribution amplitudes. Eq. (11) has the same form both for exclusive and inclusive production of ρ, but the dependence on the production angle θ is different in the two cases

$$F^{ex}(\theta) = \frac{\cos^2\theta}{1+\cos^2\theta} \tag{12}$$

$$F^{in}(\theta) = \frac{\sin^4\theta}{1+6\cos^2\theta+\cos^4\theta}\,. \tag{13}$$

Eq. (11) shows that in massless perturbative QCD only the diagonal elements of $\rho(\rho)$ survive and their values, which depend on the ρ distribution amplitudes, will be discussed in the next Section; Eq. (11) also implies that the azimuthal angular distribution (6) of the pions emitted by the ρ vector meson is flat, $W(\Phi) = 1/(2\pi)$, whereas the polar one (5) depends on $\rho_{1,1}$ (or $\rho_{0,0}$) and can be rewritten as

$$W(\Theta) = \frac{3}{2}[\rho_{1,1} + (1-3\rho_{1,1})\cos^2\Theta] \tag{14}$$

$$= \frac{3}{4}[2\rho_{0,0} + (1-3\rho_{0,0})\sin^2\Theta]\,. \tag{15}$$

III. NUMERICAL EVALUATION OF $\rho_{1,1}$ AND DEPENDENCE ON THE DISTRIBUTION AMPLITUDES

We report here for convenience from Ref. [12] the explicit expression of $|\tilde{A}_{0,0}|/|\tilde{A}_{1,-1}|$ which, together with the $F(\theta)$ functions given in Eqs. (12) and (13), enters the computation of the helicity density matrix $\rho_{1,1}$ according to Eq. (11):

$$\frac{|\tilde{A}_{0,0}|}{|\tilde{A}_{1,-1}|} = \frac{1}{\sqrt{6}}\left(\frac{f_L}{f_T}\right)^2 \frac{|I_{0,0}|}{|I_{1,-1}|} \tag{16}$$

where

$$I_{1,-1} = -\frac{1}{32}\int_0^1 dxdy\,\varphi_T(x,\tilde{Q}_x^2)\,\varphi_T(y,\tilde{Q}_y^2) \times \frac{\alpha_s[xyM_\chi^2]\,\alpha_s[(1-x)(1-y)M_\chi^2]}{xy(1-x)(1-y)(2xy-x-y)} \tag{17}$$

$$I_{0,0} = -\frac{1}{32}\int_0^1 dxdy\,\varphi_L(x,\tilde{Q}_x^2)\,\varphi_L(y,\tilde{Q}_y^2) \times \frac{\alpha_s[xyM_\chi^2]\,\alpha_s[(1-x)(1-y)M_\chi^2]}{xy(1-x)(1-y)(2xy-x-y)} \times \left[1+\frac{(x-y)^2}{2xy-x-y}\right]\,. \tag{18}$$

In the above equations M_χ is the χ_{c2} mass, $\tilde{Q}_z = \min(z, 1-z)Q$ [15] and φ_L, φ_T are the ρ distribution amplitudes which, following Refs. [1,2], are in general assumed to be different for longitudinally ($L, \lambda = 0$) or transversely ($T, \lambda = \pm 1$) polarized vector mesons; f_L and f_T are the corresponding decay constants.

To test the dependence of the numerical values of $\rho_{1,1}$ on the distribution amplitudes we have performed the computations of the integrals (17) and (18) choosing two typical different sets of φ_L and φ_T, i.e. the symmetric distribution amplitudes

$$\varphi_L(x) = \varphi_T(x) = 6x(1-x) \tag{19}$$

and the QCD sum rule ones [2]

$$\begin{aligned}\varphi_L(x,\tilde{Q}_x^2) &= 6x(1-x) \\ &\times \left\{1+\frac{1}{5}C_2^{3/2}(2x-1)\left[\frac{\alpha_s(\tilde{Q}_x^2)}{\alpha_s(\mu_L^2)}\right]^{2/3}\right\}\end{aligned} \tag{20}$$

$$\begin{aligned}\varphi_T(x,\tilde{Q}_x^2) &= 6x(1-x)\left\{\left[\frac{\alpha_s(\tilde{Q}_x^2)}{\alpha_s(\mu_T^2)}\right]^{4/25}\right. \\ &- \left.\frac{1}{6}C_2^{3/2}(2x-1)\left[\frac{\alpha_s(\tilde{Q}_x^2)}{\alpha_s(\mu_T^2)}\right]^{52/75}\right\}\end{aligned} \tag{21}$$

where $\mu_L^2 = 0.5$ (GeV$/c)^2$, $\mu_T^2 = 0.25$ (GeV$/c)^2$ [2], and $C(z)$ denotes Gegenbauer polynomials. In both cases we have $f_L = f_T$.

The latter set includes explicitly the QCD Q^2 evolution of the distribution amplitudes [1,16] and the strong coupling constant has a smooth limited small Q^2 behaviour [17]

$$\alpha_s(Q^2) = \frac{12\pi}{25\,\ln\left[(Q^2+4m_g^2)/\Lambda^2\right]} \tag{22}$$

with $m_g = 0.5$ GeV$/c$ and $\Lambda = 0.2$ GeV$/c$. The effective gluon mass m_g, which avoids enhancements in the soft end point regions, models in a convenient way the effects of Sudakov form factors [7].

Notice that Eqs. (20) and (21), although they imply the usual asymptotic ($Q^2 \to \infty$) behaviour [1]

$$\varphi_T(x,Q^2) = \varphi_L(x,Q^2)\left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu_T^2)}\right]^{4/25}, \tag{23}$$

lead, at the moderate Q^2 values involved in the processes considered here, to longitudinal and transverse distribution amplitudes which strongly differ in their x dependences; this feature strongly affects the numerical results.

The set (19), on the contrary, has the same x dependence both for φ_L and φ_T and no Q^2 evolution; the same x dependence is the dominant feature and we have checked that by adding to φ_T a Q^2 dependent factor in agreement with Eq. (23) the numerical results for the helicity density matrix do not change significantly. This is the reason why we have only considered the two choices of φ_L and φ_T given above: they are representative of whole classes of distribution amplitudes.

In Figs. 1 and 2 we show the resulting values of $\rho_{1,1}(\rho)$ as a function of the production angle θ for the two choices of the distribution amplitudes (19) and (20, 21) and, respectively,

for the exclusive and inclusive production processes. In both cases the two sets of distribution amplitudes lead to clearly different results. Notice that $\rho_{1,1}(\theta)$ is symmetric around $\theta = \pi/2$, see Eqs. (12) and (13).

The values of $\rho_{1,1}$ fix the shape of the polar angle distribution (14) of the pion resulting from the ρ decay, which is shown in Figs. 3 and 4, respectively for ρ vector particles produced at $\theta = 50°$ in the exclusive case and $\theta = 90°$ in the inclusive one. We have chosen such angles because in these kinematical regions the mass corrections are negligible and cannot affect our study of the distribution amplitudes; this can be seen from the results of Refs. [12,13] (which, however, were derived with a fixed coupling constant and no Q^2 evolution of the distribution amplitudes) and has also been explicitly verified with the present improvement of the calculations. Again, the two choices of distribution amplitudes, both in the exclusive and the inclusive channels, lead to very different pion angular distributions, whose observation would allow to discriminate between them.

Our study shows that different distribution amplitudes give different results for the diagonal elements of the ρ vector meson helicity density matrix; the ρ's are decay product of charmonium states χ_{c2}, either exclusively or inclusively produced. A measurement of $\rho_{1,1}$ is difficult and might not allow a detailed comparison between similar distribution amplitudes, but certainly allows to discriminate symmetric distribution amplitudes, Eq. (19), or other distribution amplitudes with $\varphi_L \sim \varphi_T$, from those similar to the ones inspired by QCD sum rules, Eqs. (20, 21). Even for such a qualitative information is worth attempting the measurements we propose here; the more we know about subtle hadronic properties and the more we gain in predictive power.

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FIGURES

FIG. 1. Values of $\rho_{1,1}(\rho)$ as a function of the ρ meson production angle θ, in the exclusive process (1). Solid curve: symmetric distribution amplitudes, Eq. (19); dashed curve: QCD sum rule distribution amplitudes, Eqs. (20, 21).

FIG. 2. Values of $\rho_{1,1}(\rho)$ as a function of the ρ meson production angle θ, in the inclusive process (1). Same notations for the curves as in Fig. 1.

FIG. 3. Plot of the angular distribution, $W(\Theta)$, of the π emitted in the ρ decay. The ρ has been produced in the exclusive channel at an angle $\theta = 50^\circ$. Same notations for the curves as in Fig. 1.

FIG. 4. Plot of the angular distribution, $W(\Theta)$, of the π emitted in the ρ decay. The ρ has been produced in the inclusive channel at an angle $\theta = 90^\circ$. Same notations for the curves as in Fig. 1.

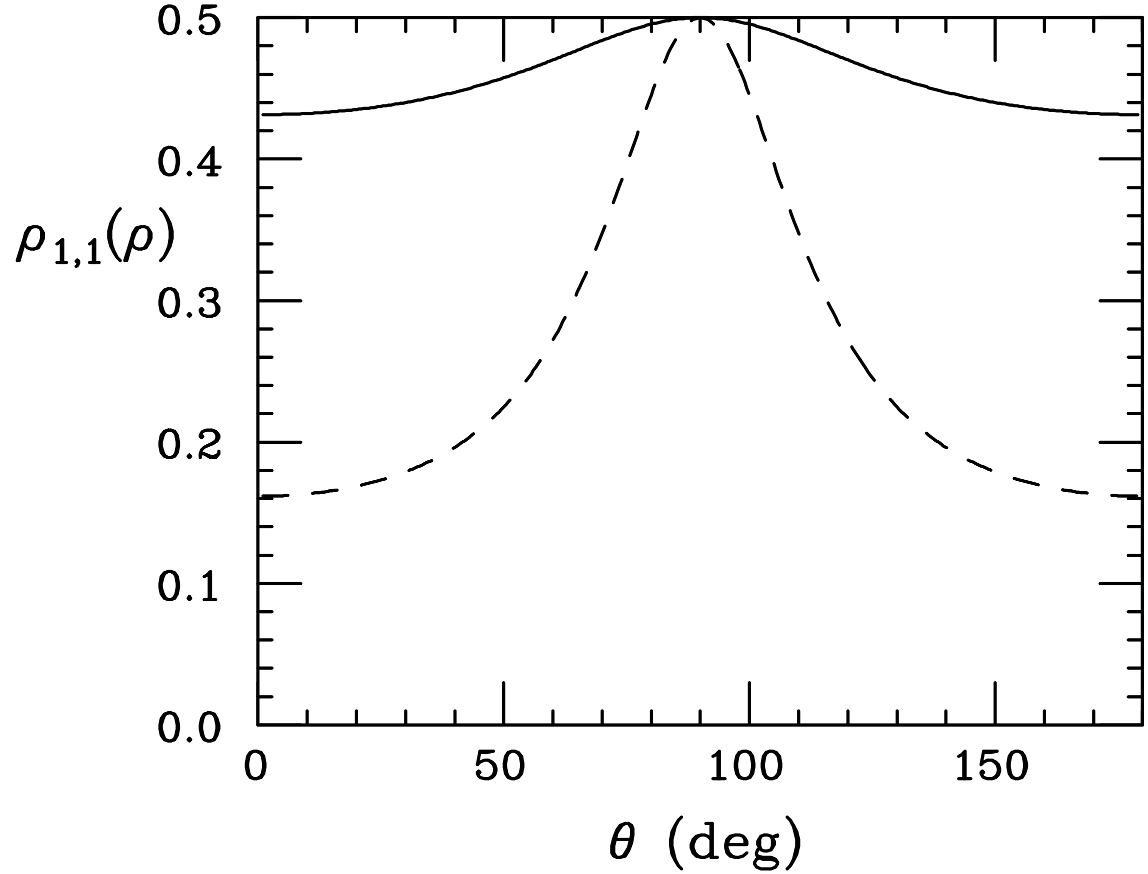

FIG. 1

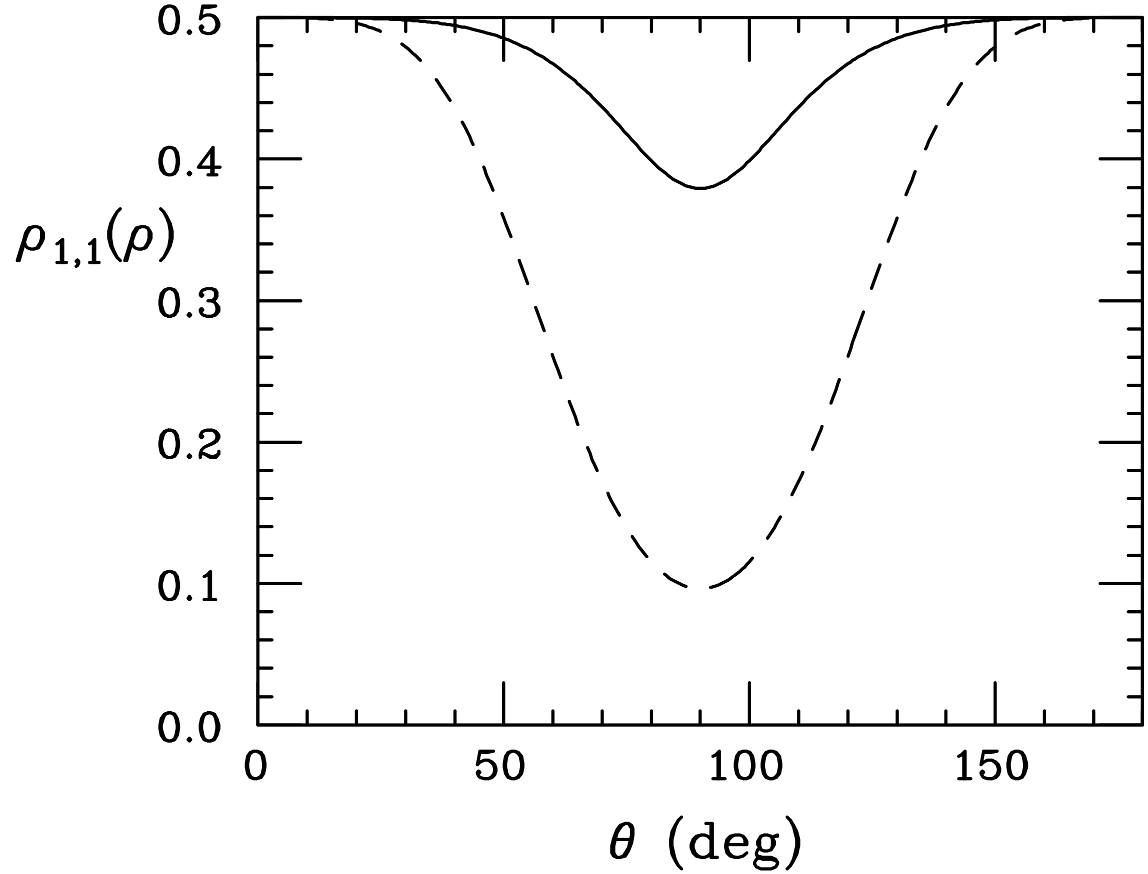

FIG. 2

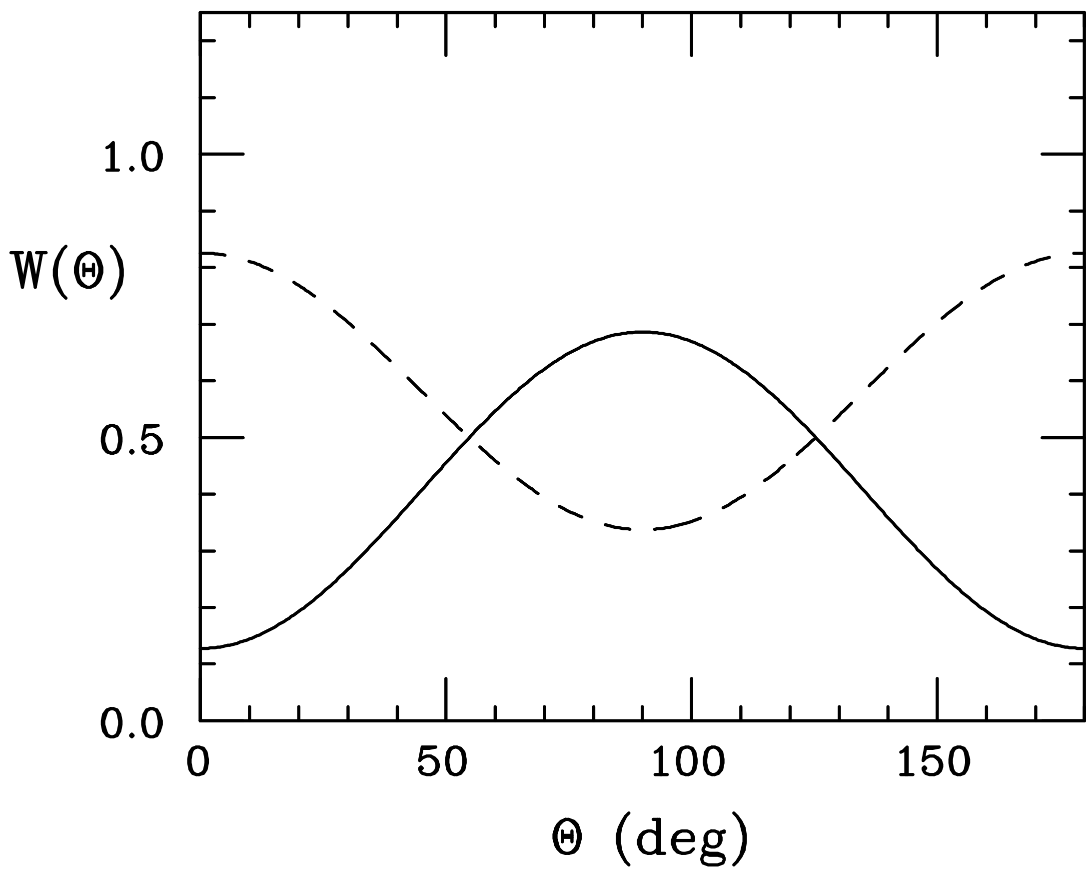

FIG. 3

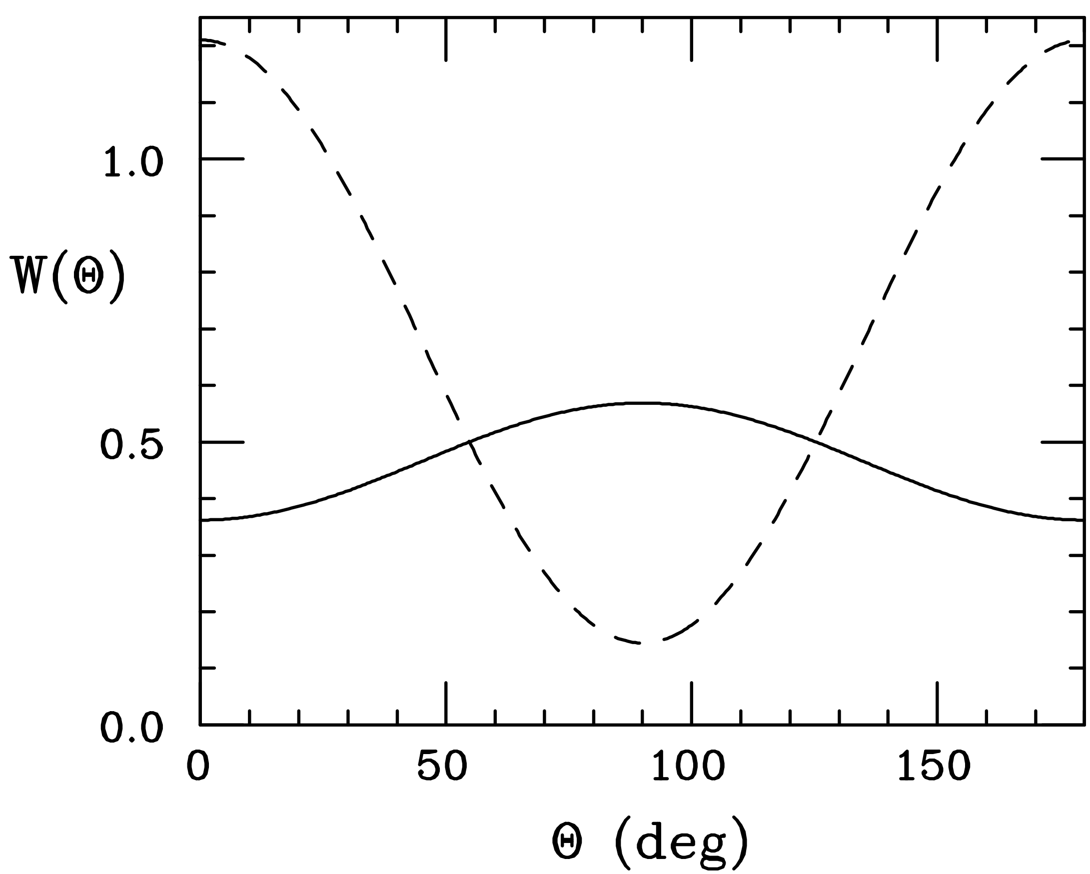

FIG. 4